SRI VENKATESWARA INTERNSHIP PROGRAM
FOR RESEARCH IN ACADEMICS
(SRI-VIPRA)

SRI-VIPRA

Project Report of 2024: SVP-2404

“Fluid Flows”

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SRIVIPRA PROJECT 2024

Title : Fluid Flows

Name of Mentor: Dr. P. Devaki **Name of Department: Mathematics** **Designation: Assistant Professor**	

List of students under the SRIVIPRA Project

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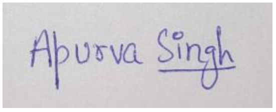
1		Apurva Singh	1722045	B. Sc. (Hons.) Mathematics	
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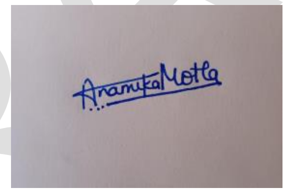
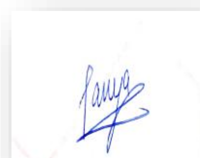
4		Anamika Motla	1723131	B. Sc. (H) Mathematics	
5		Anjali Singh	1722107	B.Sc(H)Mathematics	
6		Tanya	1723003	B.Sc(H)Mathematics	

Devaki. P

Signature of Mentor

Certificate of Originality

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project SVP-2404 titled **"Fluid Flows"**. The participants have carried out the research project work under my guidance and supervision from 1st July, 2024 to 30th September 2024. The work carried out is original and carried out in an online/offline/hybrid mode.

Devaki.P

Signature of Mentor

Acknowledgements

We would like to express our profound appreciation to all those who contributed to the successful completion of our project. Our deepest gratitude goes to our esteemed supervisor, **Dr. P. Devaki**, whose invaluable guidance, expertise, and unwavering support were pivotal throughout this research journey. Her insightful feedback and encouragement inspired us to strive for excellence.

Additionally, we would like to extend our gratitude to our honourable principal, **Prof. Vajala Ravi** for providing resources and facilities, which played a crucial role in facilitating our work.

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STUDY ON A RESEARCH PAPER

The project begins with a Study on "**Analysis on research paper "an analytical solution for fully developed forced convection in triangular ducts**". This paper presents an exact analytical solution for fully developed convective heat transfer in equilateral triangular ducts under constant heat flux at the walls. Here, the finite series expansion method is used to derive the closed form of dimensionless temperature distribution.

The paper begins with continuity, momentum and energy equations of incompressible fluid flow and heat transfer in a duct with equilateral triangular cross section. Then non dimensional parameters are introduced and dimensionless form of heat transfer equation and main flow velocity equation are obtained. Substituting dimensionless form of main flow velocity into dimensionless form of heat transfer equation, the heat transfer equation of a fully developed flow in a straight duct with equilateral triangular cross section is obtained. Then a fifth order finite series is considered for temperature distribution and substituted to heat transfer equation, same order terms are compared and unknown coefficients are achieved. Finally, non-dimensional temperature profile is obtained and graphs are plotted using mathematica.

In this paper the non-dimensional temperature profile is obtained as

$$T(y,z) = \frac{10}{27}\left(z - \frac{3}{2}\right)(9y^4 + 6y^2z^2 - 18y^2z - 3z^4 + 6z^3)$$

Using the obtained temperature distribution, the Nusselt number is calculated as 28/9.

EXTENSION OF THE RESEARCH PAPER

The above studied paper was extended for an elliptical duct instead of triangular duct.

The governing energy equations and the non-dimensional parameters of the problem can be expressed same as the base paper. In addition to that, we have:

$$a = \frac{\tilde{a}}{d_h}, \quad b = \frac{\tilde{b}}{d_h}, \quad \eta = \frac{\tilde{b}}{\tilde{a}} = \frac{b}{a} \qquad (1)$$

where ã & b̃ are the semi-major and semi-minor axes of the elliptical cross section and aspect ratio (η). The hydraulic diameter d_h:

$$d_h = \frac{4\tilde{A}}{\tilde{P}} = \frac{\pi\tilde{b}}{E(e)} \qquad (2)$$

where 'e' is the eccentricity of the ellipse. The function E is the complete elliptical integral:

$$E(e) = \int_0^{\pi/2} \sqrt{1 - e^2 sin^2\theta} d\theta$$

Further, following the same procedure as done in base paper, we obtained the main flow velocity and flow rate of rectilinear flow in duct with elliptical cross-section as

$$\tilde{u}(y,z) = \frac{1}{2\mu}\left(-\frac{d\tilde{p}}{d\tilde{x}}\right)\left(\frac{\widetilde{a^2}\widetilde{b^2}}{\widetilde{a^2} + \widetilde{b^2}}\right)\left(1 - \frac{\widetilde{y^2}}{\widetilde{a^2}} - \frac{\widetilde{z^2}}{\widetilde{b^2}}\right),$$

$$\tilde{Q} = \frac{\pi}{4\mu}\left(-\frac{d\tilde{p}}{d\tilde{x}}\right)\left(\frac{\widetilde{a^3}\widetilde{b^3}}{\widetilde{a^2} + \widetilde{b^2}}\right)$$

The bulk velocity ($\tilde{u}_b$) can be obtained as:

$$\widetilde{u_b} = \frac{\tilde{Q}}{A} = \frac{1}{4\mu}\left(-\frac{d\tilde{p}}{d\tilde{x}}\right)\frac{\tilde{a}^2\tilde{b}^2}{\widetilde{a^2} + \widetilde{b^2}}$$

Based on the above two velocity equations, the main flow velocity can be expressed as

$$u = 2\left(1 - \frac{y^2}{a^2} - \frac{z^2}{b^2}\right)$$

The heat transfer equation of fully developed flow in a straight elliptical pipe is obtained:

$$\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 8\left(1 - \frac{y^2}{a^2} - \frac{z^2}{b^2}\right)$$

Based on the definition of dimensionless temperature, the boundary condition

at $\quad \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1; \quad T = 0$

CLOSED FORM SOLUTION

We assume the temperature distribution T(y, z) as follows:

$$T(y,z) = (C_1y^4 + C_2z^4 + C_3y^2 + C_4z^2 + C_5y^2z^2 + C_6)$$

Based on the Eqs. (15) and (16), we obtain:

$$a^2 = \frac{\widetilde{a^2}}{d_h^2} = \left(\frac{E(e)}{\pi}\right)^2 \frac{1}{\eta^2}, \quad b^2 = \frac{\widetilde{b^2}}{d_h^2} = \left(\frac{E(e)}{\pi}\right)^2$$

After finding the values of unknown coefficients the final non-dimensional temperature is

$$T(y,z) = -2\left(\frac{\pi}{E(e)}\right)^2 \left(\frac{1}{3(\eta^6 + 7\eta^4 + 7\eta^2 + 1)}\right) \{\eta^4(\eta^4 + 6\eta^2 + 5)y^4 + (5\eta^4 + 6\eta^2 + 1)z^4 - 2\left(\frac{E(e)}{\pi}\right)^2 [\eta^2(3\eta^4 + 16\eta^2 + 5)y^2 + (5\eta^4 + 16\eta^2 + 3)z^2] + 6\eta^2(\eta^4 + 2\eta^2 + 1)y^2z^2 + \left(\frac{E(e)}{\pi}\right)^4 (5\eta^4 + 26\eta^2 + 5)\}$$

By substituting y = 0 and z = 0 in above equation, we get the value of dimensionless temperature at the center of cross section as:

$$T_c = -\left(\frac{E(e)}{\pi}\right)^2 \frac{2(5\eta^4 + 26\eta^2 + 5)}{3(\eta^4 + 7\eta^2 + 7\eta^2 + 1)}$$

The value of E(e) can be approximately calculated as

$$E(e) \approx \frac{\pi}{4}(\eta + 1)\left[1 + \frac{3s}{10 + \sqrt{4 - 3s}}\right], \quad \text{where} \quad s = \frac{(1-\eta)^2}{(1+\eta)^2}$$

Thus, the approximate value of T_c obtained as

$$T_c \approx \frac{(5\eta^4 + 26\eta^2 + 5)(\eta + 1)^2}{24(\eta^4 + 7\eta^2 + 7\eta^2 + 1)} \times \left[1 + \frac{3s}{10 + \sqrt{4 - 3s}}\right]^2$$

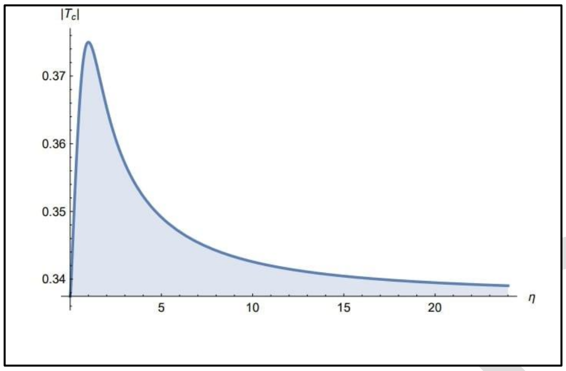

Figure 3: Variation of absolute value of temperature at the centre of elliptical duct with the aspect ratio

In conclusion, the insights gained from this paper reaffirm the critical role of exact analytical solution. Further extensions can be done by introducing Q is heat source effects.